

A simple approach for predicting the ultimate lateral capacity of a rigid pile in sand

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Rigid piles are widely used as foundations to resist lateral load. A calculation to confirm that the ultimate lateral geotechnical capacity of such piles is not exceeded is required for ultimate limit-state design. Existing (empirical) approaches employed to predict the lateral capacity of a rigid pile in sand assume the net soil pressure is a function of the vertical effective stress and friction angle. However, difficulties in determining the appropriate friction angle(s) make these approaches less useful for practical purposes. A simple alternative approach correlated with the cone penetration test end resistance (q_c) is developed in this paper based on ultimate capacities measured in a new series of tests on rigid piles with diameters of 127, 169, 273 and 457 mm and aspect ratios (L/D) between 1.6 and 12. These test results are combined with additional tests reported in the literature to establish a simple formulation for the ultimate net pressure adjacent to a pile in sand. Explicit design equations are also presented for soil profiles with a constant q_c value and where q_c increases linearly with depth. The proposed equations are seen to provide good estimates of capacities for rigid piles with a wide range of diameters.

KEYWORDS: bearing capacity; field instrumentation; piles & piling

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NOTATION

D	pile diameter
D_{50}	mean effective particle size
D_r	soil relative density
d	depth of rotation centre
e	loading height
H	lateral force at pile head
H_b	force at pile base
H_u	ultimate lateral force
L	pile embedded length
M_b	moment at pile base
M_u	ultimate lateral moment
P	lateral soil pressure
P_u	ultimate lateral soil pressure
p_a	atmospheric pressure
q_c	cone penetration test (CPT) cone tip resistance
t	pile wall thickness
y	soil lateral displacement, pile deflection
z	depth
σ'_v	vertical effective stress

INTRODUCTION

Ultimate limit-state (ULS) pile design requires a check to confirm that the ultimate lateral geotechnical capacity of the pile is not exceeded. Consequently, many (empirical) models

have been proposed to predict the lateral bearing capacity of rigid piles in (uniform) drained soils (e.g. Brinch Hansen, 1961; Broms, 1964; Petrasovits & Award, 1972; Prasad & Chari, 1999). Figure 1 presents a widely adopted model proposed by Petrasovits & Award (1972) where the pile is assumed to rotate about a point at a depth d below the ground surface. The ultimate soil pressure, P_u , at any point along the pile is generally assumed to be proportional to the vertical effective stress (σ'_v) by a constant K that is related to the plane strain coefficient of passive pressure (K_p) – for example, $K = 3K_p$ (Broms, 1964) and $K = K_p^2$ (Barton, 1982). Reese *et al.* (1974) and API (2014) propose K as a function of the normalised depth, z/D . A soil reaction shear force ('base shear', H_b) and base moment (M_b) can also be added at the pile tip resulting from the rigid rotation of the pile.

For a given loading eccentricity $e = M_u/H_u$, the lateral pile capacity H_u can be calculated with the following equations based on force and moment equilibrium:

$$H_u + H_b = \int_0^d P_u D dz - \int_d^L P_u D dz \quad (1)$$

$$M_u - H_b L - M_b = \int_0^d P_u D z dz - \int_d^L P_u D z dz \quad (2)$$

where H_u is the ultimate lateral force at the soil surface, M_u is the moment at the soil surface ($M_u = H_u \times e$), H_b is the shear force at pile tip, D is the pile diameter, L is the pile length, d is the depth to the rotation centre and P_u is the ultimate net soil pressure. Based on field tests and numerical simulations, Wang *et al.* (2020) found the relative influence on H_u of the pile base for all pile diameters is negligible. Numerical analyses presented by Burd *et al.* (2020) also indicate that the effect of base resistance is small even for piles with $L/D = 2$, although the results suggested that distributed moment (arising due to friction on the pile shaft) had to be considered alongside the contributions from

Manuscript received 24 January 2020; first decision 30 June 2020; accepted 30 June 2020.

Published online at www.geotechniqueletters.com on 31 July 2020.

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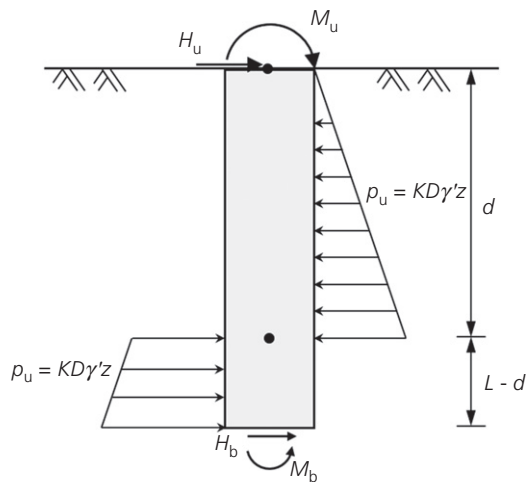


Fig. 1. Simplified model of soil–pile interaction mechanism

lateral soil resistance. As p – y curves used in general practice are derived by double differentiation of the measured bending moment profile (therefore implicitly incorporating the distributed moment), the analysis presented in this paper sets the base contribution to zero and combines the contribution of lateral soil resistance and the distributed moment components into the resultant lateral soil resistance.

Given that undisturbed sand samples are rarely recovered and that laboratory samples are not subjected to the modes of shearing imposed by laterally loaded piles, the assessment of an appropriate sand friction angle and hence P_u is essentially an engineering judgement. Consequently, cone penetration test (CPT)-based design approaches are preferred, particularly due to the correspondence observed between P_u and CPT end resistance (q_c), such as given by the following expression presented by Suryasentana & Lehane (2014):

$$P_u = 2.4q_c^{0.67}\sigma_v^{0.33}\left(\frac{z}{D}\right)^{0.75} \quad (3)$$

The study presented in this paper builds on this previous research to develop a CPT-based approach to predict the lateral capacity of a rigid pile. The lateral capacities measured in a new series of laterally loaded pile tests conducted at a sand site in Perth, Australia are reported. These tests were performed on piles with a wide range of diameters and aspect ratios. The results are combined with additional tests reported in the literature and with insights from numerical research to establish a simple formulation for the ultimate net pressure adjacent to a pile in sand as well as expressions for lateral pile capacity in idealised soil profiles.

SHENTON PARK FIELD TESTS

Geotechnical conditions

The new series of lateral load tests performed for this study was performed at the University of Western Australia (UWA) Shenton Park field test site in Perth. The site has been used extensively for foundation testing research (Xu, 2007; Lehane, 2008; Li & Lehane, 2010) and the ground conditions have been investigated systematically in Lehane *et al.* (2004). The soil stratigraphy at this site consists of a 5–10 m thick deposit of aeolian siliceous sand with a minor carbonate content (<5%) overlying a limestone formation.

The water table is at about 15 m depth within the limestone layer. The effective particle sizes of the sand, D_{50} , D_{60} and D_{10} are about 0.42, 0.47 and 0.21 mm, respectively. The maximum and minimum void ratios are 0.81 and 0.45, respectively, the average bulk density is 1670 kg/m³ and the degree of saturation is less than 15%.

The site area in the vicinity of the pile tests was lowered by 0.5 m to eliminate the influence of a slightly variable upper crust on the piles' lateral responses. CPTs performed in the excavated area are plotted in Fig. 2 and are consistent with a uniform relative density (D_r) of 64%; this value was derived from the equation provided in this figure and developed by Xu (2007) for the Shenton Park site. The low q_c values measured in the upper 250 mm of the site (equivalent to seven cone diameters) arise due to shallow penetration effects and therefore 'steady state' q_c values equivalent to those determined for $D_r = 64\%$ are employed at these depths in the analysis presented below.

Test details

Eleven static lateral pile load tests, with details summarised in Table 1, were performed for this study. Four different diameter piles were employed, namely 127, 169, 273 and 457 mm. The lateral capacity for each diameter pile was measured for three different embedded lengths (i.e. 0.75, 1.0 and 1.5 m) except for the 169 mm pile with two embedded lengths (i.e. 1.0 and 1.5 m). This test programme investigated aspect ratios varying from 1.6 to 11.8, which covers the range adopted by monopiles used in existing offshore wind turbine projects.

The tests were conducted by pushing pairs of piles apart using a 50 kN capacity load cell, a rigid steel tube and a hydraulic jack. Two curved wooden blocks formed platens at the connections between the loading system and the piles. The horizontal load was applied 0.34 ± 0.02 m above the ground surface for all tests. Pile deflections above ground level were measured using string pot (wire) sensors, which were fixed on a reference frame at known positions along the piles above the ground surface.

Load–deflection response at the ground surface

The load–deflection responses measured for each diameter pile are presented in Fig. 3. As shown in the figure, the reaction force for each diameter pile increased with pile embedded length and the load–deflection curves exhibit continuous hardening with increasing pile deflection.

The method first suggested by Manoliu *et al.* (1985) was adopted to provide a consistent means of defining ultimate capacities (H_u) for all the test piles. H_u values were obtained by determining the best-fit line to the y against y/H data. These are summarised in Table 1 and the associated best-fit hyperbolic curves are plotted in Fig. 3. It is evident that the pile load–deflection responses are well matched by the hyperbolic form.

FIELD TESTS REPORTED IN THE LITERATURE

Previously published results for laterally loaded piles in sand were collected to expand the database from the Shenton Park study. Three well-documented case histories presented by Murphy *et al.* (2018), Li *et al.* (2014) and McAdam *et al.* (2019) were selected and cover a range of diameters of 245 mm to 2 m and a range of pile lengths from 2.2–10.6 m. All of the 16 piles considered responded in a rigid manner to the applied lateral load.

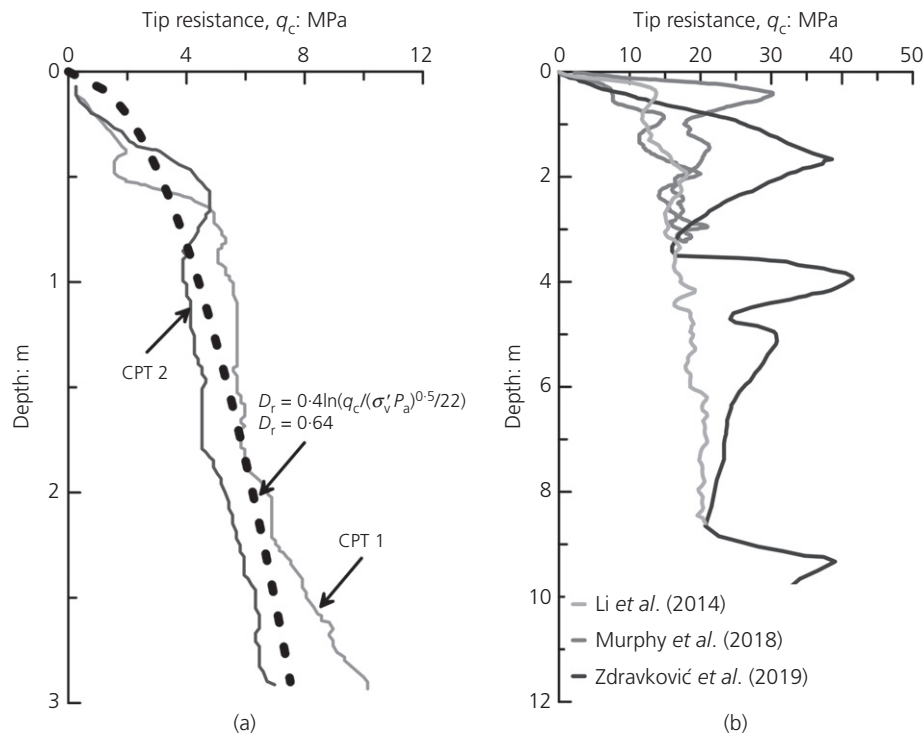


Fig. 2. CPT profile of: (a) this study; (b) those in literature (Note: the profile of q_c in McAdam *et al.* (2019) is the mean profile from Zdravković *et al.* (2019))

Table 1. Summary of the field tests of this study and those in the literature

Test ID	D : m	L : m	L/D	t : m	e : m	$H_{ultimate}$: kN	C
This study							
1	0.127	0.75	5.91	0.019	0.34	6.66	0.49
2	0.127	1.00	7.87	0.019	0.32	9.64	0.41
3	0.127	1.50	11.81	0.019	0.36	20.88	0.45
4	0.169	1.00	5.92	0.0045	0.34	9.13	0.30
5	0.169	1.50	8.88	0.0045	0.33	20.20	0.32
6	0.273	0.75	2.75	0.0064	0.34	9.65	0.32
7	0.273	1.00	3.66	0.0064	0.33	15.04	0.30
8	0.273	1.50	5.49	0.0064	0.35	42.02	0.42
9	0.457	0.75	1.64	0.0064	0.35	13.68	0.28
10	0.457	1.00	2.19	0.0064	0.34	24.15	0.29
11	0.457	1.50	3.28	0.0064	0.34	45.25	0.27
Murphy <i>et al.</i> (2018)							
12	0.245	1.50	6.12	0.008	0.40	49.75	0.30
13	0.51	1.50	2.94	0.01	1.00	107.53	0.22
14	0.51	2.25	4.41	0.01	1.00	212.77	0.23
15	0.51	3.00	5.88	0.01	1.00	555.56	0.40
McAdam <i>et al.</i> (2019)							
16	0.762	2.27	2.98	0.01	9.99	54.95	0.21
17	0.762	2.24	2.94	0.01	10.00	57.47	0.23
18	0.762	3.98	5.22	0.014	9.98	277.78	0.31
19	0.762	3.96	5.20	0.014	10.00	263.16	0.30
20	0.762	6.02	7.90	0.025	10.06	769.23	0.36
21	2	10.61	5.31	0.038	9.90	5624.30	0.37
22	2	10.57	5.29	0.038	9.89	5656.11	0.38
Li <i>et al.</i> (2014)							
23	0.34	2.20	6.47	0.014	0.40	135.14	0.25
24	0.34	4.35	12.79	0.014	0.40	909.09	—
25	0.34	3.10	9.12	0.014	0.40	454.55	—
26	0.34	5.00	14.71	0.014	1.32	303.03	—
27	0.34	7.00	20.59	0.014	1.32	476.19	—

The details of the pile load tests are presented together with the Shenton Park results in Table 1 while the reported load–displacement curves are presented in Fig. 4. CPT q_c profiles at each of the sites are presented in Fig. 2(b).

EXPRESSION FOR ULTIMATE NET PRESSURE
Wesselink *et al.* (1988), Dyson & Randolph (2001) and Suryasentana & Lehane (2014, 2016) have shown that the ultimate lateral pressure varies in proportion to the q_c value

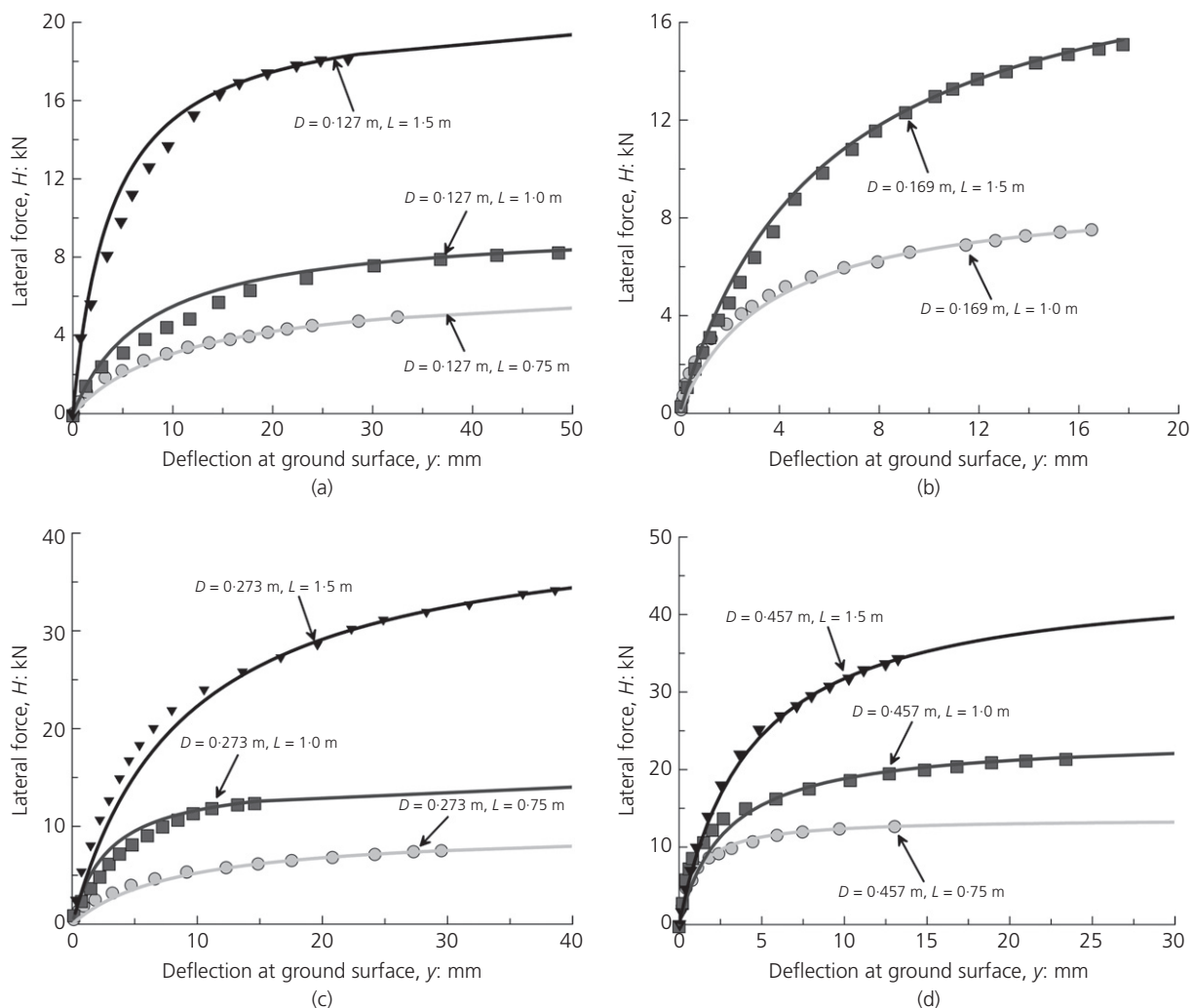


Fig. 3. Measured load–deflection response of different diameter piles: (a) $D=0.127$ m; (b) $D=0.169$ m; (c) $D=0.273$ m; (d) $D=0.457$ m (Note: the symbol represents the measured data and the line represents the fitted data with the adopted methods for ultimate bearing capacity)

raised to the power of about 0.7 (e.g. see equation (3)). Formulations proposed by these authors target typical levels of displacement under service loads (i.e. less than 10% of the pile diameter) and do not focus explicitly on ultimate conditions, which can require very large displacements (the 127 mm diameter piles in Fig. 3 develop H_u at a displacement of about $2.5D$).

A re-assessment of the database of numerical analyses reported by Suryasentana & Lehane (2014, 2016), which involved processing of results at additional depths, indicated that the effect of the normalised depth (z/D) on P_u is not as significant as that implied by equation (3) for a pile undergoing rotation and that such a dependence overestimates the z/D influence as z/D increases. The following form for P_u was therefore considered in the assessment of the database of lateral pile tests:

$$P_u = Cq_c^a \sigma_v^b p_a^{1-a-b} \quad (4)$$

where C , a and b are the empirical constants and p_a is the atmospheric pressure (100 kPa).

Equation (4) was employed together with equations (1) and (2) and the q_c profiles in Fig. 2 to determine the empirical constants in equation (4) that gave a best fit to the H_u values determined in all 27 load tests summarised in Table 1. An ‘ a ’ value of 0.7 was found to be appropriate and

in line with existing formulations. The corresponding C values obtained for $b=0.1$ are plotted against pile diameter in Fig. 5 and encouragingly show that the value of C is comparable at all sand sites and pile configurations considered. Apart from the higher C values recorded on the small 127 mm diameter piles (for which corrections for shallow penetration effects were significant; see Fig. 2), the value of C averages at 0.3 with a relatively low coefficient of variation of 0.2. The following expression for P_u is therefore recommended for determination of ultimate lateral pile capacity:

$$P_u = 0.3q_c^{0.7} \sigma_v^{0.1} p_a^{0.2} \quad (5)$$

EXPRESSIONS FOR ULTIMATE LATERAL PILE CAPACITY

Equation (5) can be employed to determine explicit expressions for the ultimate lateral capacity (H_u) of a rigid pile in sand deposits with idealised q_c profiles, assuming an average submerged unit weight along the pile of γ' . Two common cases are examined, namely where (a) q_c increases linearly with depth and (b) q_c is constant with depth.

(a) q_c increasing linearly with depth

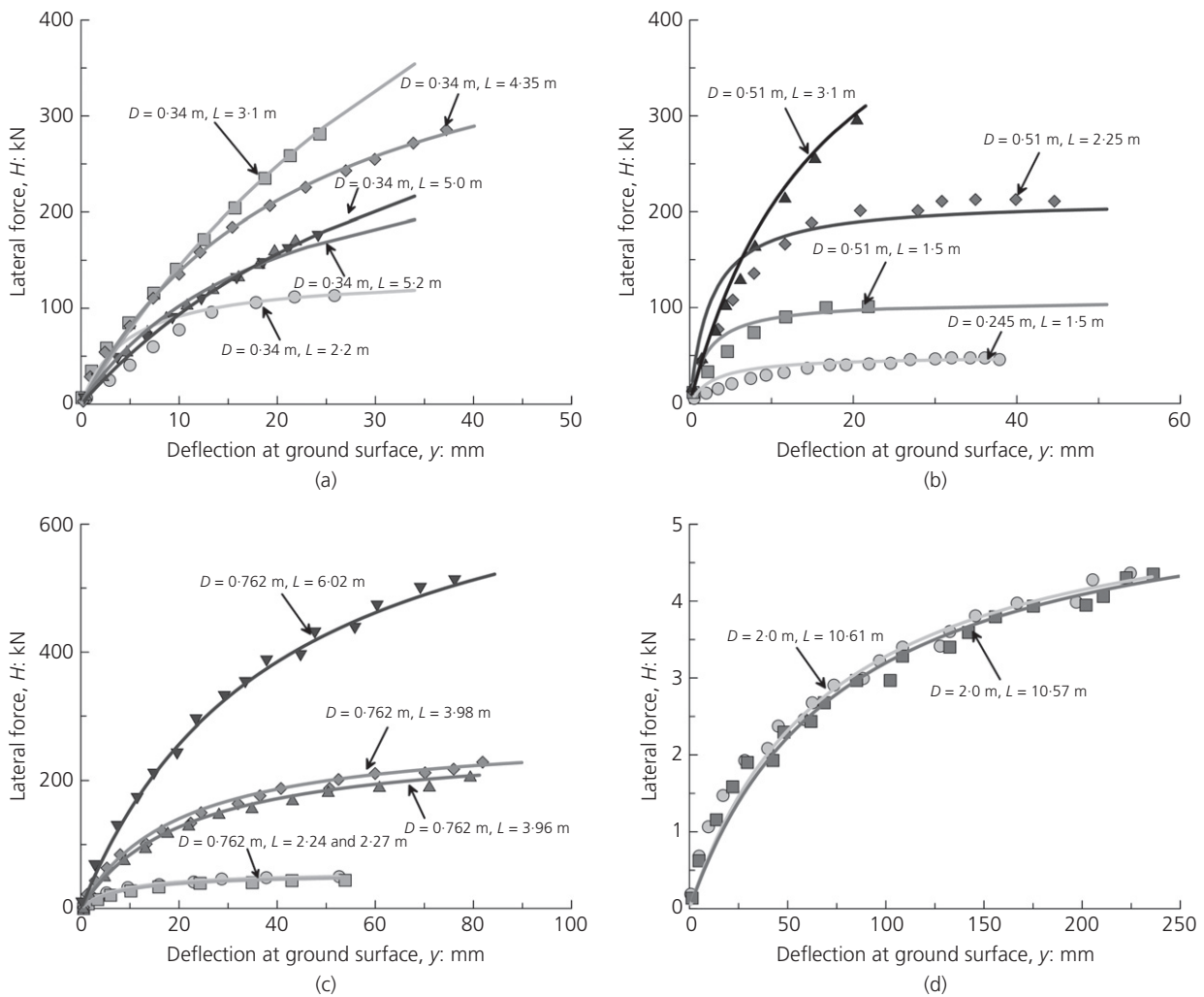


Fig. 4. Field test load–deflection response in: (a) Li *et al.* (2014); (b) Murphy *et al.* (2018); (c) McAdam *et al.* (2019); (d) McAdam *et al.* (2019)

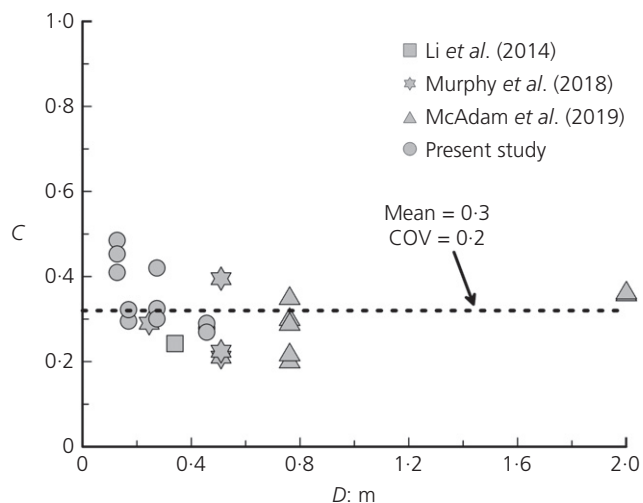


Fig. 5. Empirical constants C

When q_c increases linearly with depth (z) with a gradient q'_c ,

$$q_c = q'_c z \tag{6}$$

$$P_u = 0.3(q'_c z)^{0.7} (y'z)^{0.1} p_a^{0.2} \tag{7}$$

Combining equations (1), (2) and (7) and ignoring any contribution of the base (as discussed), it can be shown that the depth to the rotation centre (d) and the loading eccentricity (e) have the following relationship:

$$\eta = \frac{e}{L} = \frac{1.8(1 - 2\alpha^{2.8})}{2.8(2\alpha^{1.8} - 1)} \tag{8a}$$

$$\text{where } \alpha = \frac{d}{L} \tag{8b}$$

Equations (1), (2), (7) and (8) then lead to the following expression for the ultimate lateral load, H_u :

$$H_u = C_1 DL (q'_c L)^{0.7} (y' L)^{0.1} p_a^{0.2} \tag{9a}$$

$$\text{where } C_1 = 0.167(2\alpha^{1.8} - 1) \tag{9b}$$

By solving equation (8a) for different e/L ratios, it was found that the following equation provides a good match to the relationship between the C_1 and loading eccentricity ratio (e/L):

$$C_1 = \frac{0.05}{1 + ((e/L)/0.745)} \tag{9c}$$

(b) q_c constant

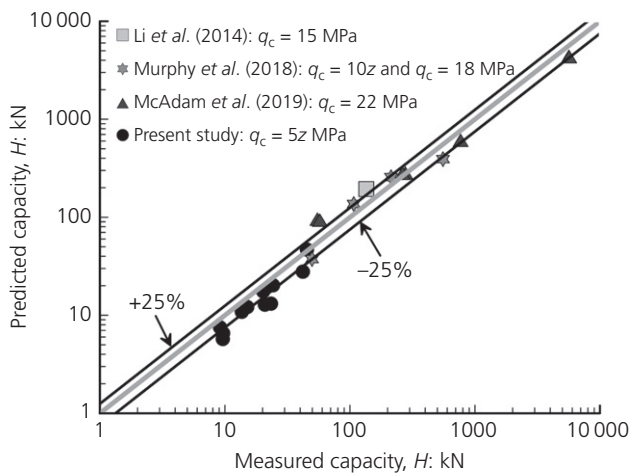


Fig. 6. Measured and calculated lateral capacity with proposed explicit equations

When q_c is constant with depth, the lateral capacity can be derived from equations (1), (2) and (7) as

$$H_u = C_2 DL (q_c)^{0.7} (\gamma' L)^{0.1} p_a^{0.2} \quad (10a)$$

The constant C_2 varies with e/L (as for C_1 in equation (9)). The best-fit relationship obtained for C_2 is as follows:

$$C_2 = \frac{0.115}{1 + ((e/L)/0.65)} \quad (10b)$$

The ultimate lateral capacity (H_u) of a rigid pile can therefore be obtained simply from equation (9) when q_c increases linearly with depth and equation (10) when q_c is constant. Figure 6 presents a comparison between the measured lateral capacities for the pile tests summarised in Table 1 with those predicted using the relevant equations corresponding to the specific q_c profile at each site (see Fig. 2). It is seen that H_u values (which vary by three orders of magnitude) are generally predicted to within 25% using these simplified expressions.

CONCLUSIONS

The results from a series of lateral load tests conducted on short rigid piles are combined with other field test data reported in the literature to derive a simple expression for the ultimate lateral net stress described as a function of the CPT end resistance (q_c). This expression is then used to develop simplified equations that foundation designers can employ to determine an approximate estimate of the ultimate lateral pile capacity (H_u) in sand deposits where q_c is constant or increases linearly with depth.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support provided by National Key Research and Development Program (2018YFE0109500), National Natural Science Foundation of China (51939010, 51779221 and 51679174) and the Key Research and Development Program of Zhejiang Province (2018C03031). The first author acknowledges the support from China Scholarship Council (201806320082). The third author is the Fugro chair in Geotechnics, whose support is gratefully acknowledged.

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